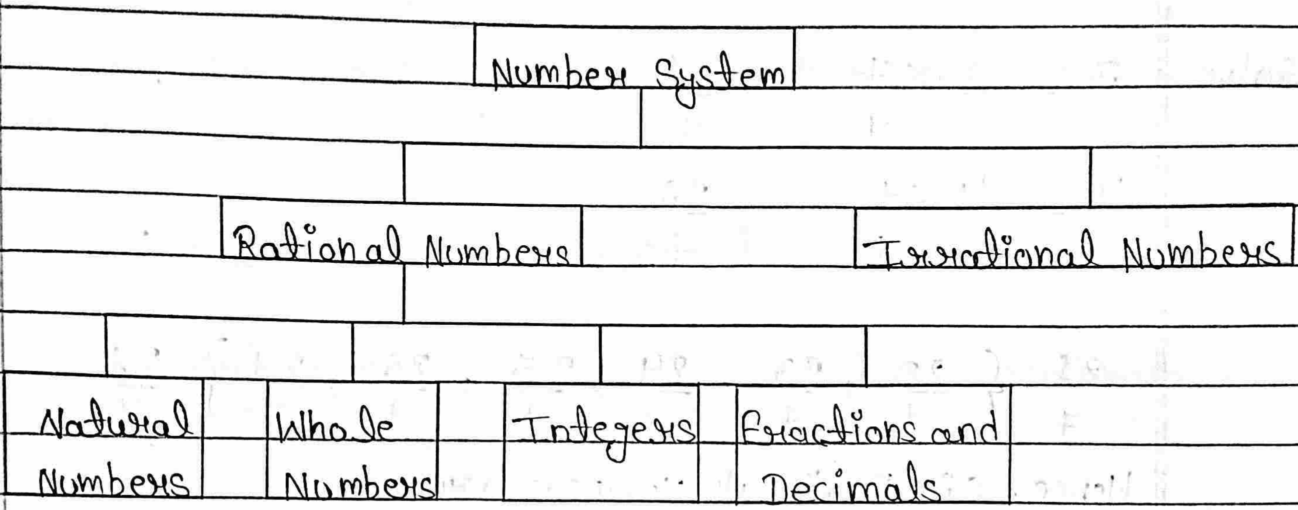
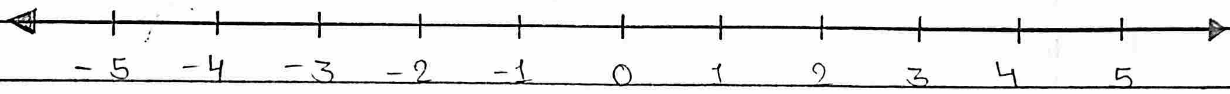
Chapter - 1

Number Systems

Ex. 1.1





~~Nuber~~ Number Line

$$\text{Rational Number} = \frac{\text{Numerator}}{\text{Denominator}} = \frac{p}{q}$$

where, $q \neq 0$

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Salve Yes, zero is a rational number. It can be written as $\frac{0}{1}, \frac{0}{2}, \frac{0}{3}$, etc., in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

2. Find six rational numbers between 3 and 4.

Solve $3 = \frac{3 \times 7}{7} = \frac{21}{7}$

$4 = \frac{4 \times 7}{7} = \frac{28}{7}$

$\frac{21}{7} \left\{ \frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7} \right\} \frac{28}{7}$

Hence, six Rational Numbers are

$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$

3. Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$

Solve $\frac{3}{5} = \frac{3 \times 6}{5 \times 6} = \frac{18}{30}$

$\frac{4}{5} = \frac{4 \times 6}{5 \quad 6} = \frac{24}{30}$

$\frac{18}{30} \left\{ \frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30} \right\} \frac{24}{30}$

4. State whether the following statements are true or false. Give reasons for your answer.

(i) Every natural number is a whole number. $\boxed{\text{True}}$

(ii) Every integer is a whole number. $\boxed{\text{False}}$

(iii) Every rational number is a whole number. $\boxed{\text{False}}$

Solve (i) True, since the collection of whole numbers contains all the natural numbers.

(ii) False, Negative integers are not whole numbers.

(iii) False, Numbers such as $\frac{2}{3}$, $\frac{3}{4}$, $\frac{-3}{5}$, etc are rational numbers but not whole numbers.

Ex. 1.2

1. State whether the ~~four~~ following statements are true or false. Justify your answers.

(i) Every irrational number is a real number.

(ii) Every point on the number line is of the form $\sqrt{m}$, where m is a natural number.

(iii) Every real number is an irrational number.

Solve (i) True. All irrational and rational numbers together make up the collection of real number.

(ii) False, example between $\sqrt{2}$ and $\sqrt{3}$ there are infinitely many numbers and these can ~~be~~ not be represented in the form $\sqrt{m}$, where m is a natural number.

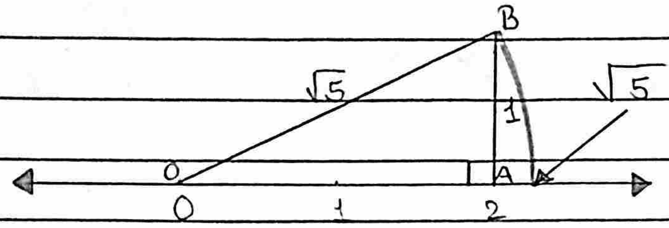
(iii) False. All rational numbers are also real numbers.

2. Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.

Solve Square roots of all positive integers are not irrational. For example $\sqrt{4} = 2$, which is a rational number.

3. Show how $\sqrt{5}$ can be represented on the number line.

Solve



In $\triangle OAB$, By using Pythagoras theorem,

$(OB)^2 = (OA)^2 + (AB)^2$

$(OB)^2 = (2)^2 + (1)^2$

$(OB)^2 = 4 + 1$

$(OB)^2 = 5$

$OB = \sqrt{5}$

Ex.1.3

1. Write the following in decimal form and say what kind of decimal expansion each has:

(i) $\frac{36}{100}$

Solve $\frac{36}{100} = 0.36$

It is a Terminating decimal expansion.

(ii) $\frac{1}{11}$

Solve $\frac{1}{11} = \frac{1 \times 100}{11 \times 100} = \frac{100}{11 \times 100} = \frac{9.0909\ldots}{100}$

$= 0.090909\ldots$

$= 0.\overline{09}$

It is a non-terminating decimal expansion.

(iii) $4\frac{1}{8}$

Solve $4\frac{1}{8} = \frac{33}{8} = 4.125$

It is a terminating decimal expansion.

(iv) $\frac{3}{13}$

Solve $\frac{3}{13} = 0.230769......$

$= 0.\overline{230769}$

It is a non-terminating decimal expansion.

(v) $\frac{2}{11}$

Solve $\frac{2}{11} = 0.18.....$

$= 0.\overline{18}$

It is a non-terminating decimal expansion.

(vi) $\frac{329}{400}$

Solve $\frac{329}{400} = \frac{329}{4 \times 100} = \frac{82.25}{100}$

$= 0.8225$

It is a terminating decimal expansion.

2. You know that $\frac{1}{7} = 0.\overline{142857}$. Can you predict what the decimal expansions of $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$ are without actually doing the long division? If so, how?

[Hint : Study the remainders while finding the value of $\frac{1}{7}$ carefully]

Solve Given that

$\frac{1}{7} = 0.\overline{142857}$

$\frac{2}{7} = \frac{2 \times 1}{7} = 2 \times 0.\overline{142857} = 0.\overline{285714}$

$\frac{3}{7} = \frac{3 \times 1}{7} = 3 \times 0.\overline{142857} = 0.\overline{428571}$

$\frac{4}{7} = \frac{4 \times 1}{7} = 4 \times 0.\overline{142857} = 0.\overline{571428}$

$\frac{5}{7} = \frac{5 \times 1}{7} = 5 \times 0.\overline{142857} = 0.\overline{714285}$

$\frac{6}{7} = \frac{6 \times 1}{7} = 6 \times 0.\overline{142857} = 0.\overline{857142}$

3. Express ~~0.99999 - - - - -~~ the following in the form $\frac{p}{q}$, ~~Are you~~ where p and q are integers and $q \neq 0$.

(i) $0.\overline{6}$

Solve Let $x = 0.\overline{6}$

$x = 0.66666 - - - - - -$

One digit 6 is repeating.

We multiply it with 10 on both sides.

$10x = 10 \times 0.66666 - - - - -$

$10x = 6.6666 - - - -$

$10x = 6.\overline{6}$

$10x = 6 + 0.\overline{6}$

$10x = 6 + x$

$10x - x = 6$

$9x = 6$

$x = \frac{6}{9} = \frac{2}{3}$

$\boxed{x = \frac{2}{3}}$

hence, $0.\overline{6} = \frac{2}{3}$

(ii) $0.4\overline{7}$

Solve Let $x = 0.4\overline{7}$

$x = 0.47777\ldots$

One digit 7 is repeating.

We multiply it with 10 on both sides.

$10x = 10 \times 0.47777\ldots$

$10x = 4.7777\ldots$

$10x = 4.\overline{7}$

Again multiply by 10 to both sides.

$10 \times 10x = 10 \times 4.7777\ldots$

$100x = 47.777\ldots$

$100x = 47.\overline{7}$

$100x = 43 + 4.\overline{7}$

$100x = 43 + 10x$

$100x - 10x = 43$

$90x = 43$

$\boxed{x = \frac{43}{90}}$

hence, $0.4\overline{7} = \frac{43}{90}$

(iii) $0.\overline{001}$

Solve Let $x = 0.\overline{001}$

$x = 0.001001001\ldots$

Three digit 001 is repeating.

We multiply it with 1000 on both sides.

$1000x = 1000 \times 0.001001001\ldots$

$1000x = 1.001001\ldots$

$1000x = 1.\overline{001}$

$1000x = 1 + 0.\overline{001}$

$1000x = 1 + x$

$1000x - x = 1$

$999x = 1$

$\boxed{x = \frac{1}{999}}$

hence, $0.\overline{001} = \frac{1}{999}$

4. Express $0.99999\ldots$ in the form $\frac{p}{q}$. Are you surprised by your answer? With your teacher and classmates discuss why the answer makes sense.

Solve Let $x = 0.99999\ldots$

$x = 0.\overline{9}$

One digit 9 is repeating.

We multiply it with 10 on both sides.

$$10x = 10 \times 0.99999\ldots$$
$$10x = 9.9999\ldots$$
$$10x = 9.\overline{9}$$
$$10x = 9 + 0.\overline{9}$$
$$10x = 9 + x$$
$$10x - x = 9$$
$$9x = 9$$
$$x = \frac{9}{9}$$
$$\boxed{x = 1}$$

The answer makes sense as $0.\overline{9}$ is infinitely close to 1, i.e., we can make the difference between 1 and 0.9999..... as small as we wish by taking enough 9's.

5. What can the maximum number of digits be in the repeating block of digits in the decimal expansion of $\frac{1}{17}$? Perform the division to check your answer.

Solve $\therefore \frac{1}{17} = 0.0588235294117647$

17) 100 (0.0588235294117647

85

150

136

X140

136

XX40

34

X60

51

X90

85

X50

34

160

153

XX70

68

X20

17

X30

17

130

119

X110

102

X80

68

120

119

XX1

hence, The maximum number of digits be in the repeating block of 16 digits in the decimal expansion of $\frac{1}{17}$.

6. Look at several examples of rational numbers in the form $\frac{p}{q}$ $(q \neq 0)$, where p and q are integers with no common factors other than 1 and having terminating decimal representations (expansions). Can you guess what property q must satisfy?

Solve The prime factorisation of q has only powers of 2 or powers of 5 or both.

7. Write three numbers whose decimal expansions are non-terminating non-recurring.

Solve Three non-terminating non-recurring decimal expansion are

0.0100100010000 1-------

0.2020020002 00002-----

0.003000300003------

8. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$.

Solve $\frac{5}{7} = 0.\overline{714285}$

$\frac{9}{11} = 0.\overline{81}$

Three irrational number are

0.72010010002 -----
0.73010010002 -----
0.74010010002 -----

9. Classify the following numbers as rational or irrational:

(i) $\sqrt{23}$ (ii) $\sqrt{225}$

Solve $\sqrt{23}$ = Irrational Number

Solve $\sqrt{225} = \sqrt{15 \times 15} = 15$
= Rational Number

(iii) 0.3796 (iv) 7.478478 ----

Solve 0.3796 = Rational Number

Solve $\Rightarrow$ 7.478478 ----
$\Rightarrow 7.\overline{478}$ = irrational Number

(v) 1.101001000100001 -----

Solve 1.101001000100001 ------ = irrational Number

Ex. 1.4

1. Classify the following numbers as rational or irrational:

(i) $2 - \sqrt{5}$

Solve where, 2 = Rational

$\sqrt{5}$ = Irrational

$\therefore 2 - \sqrt{5}$ = Irrational

(iii) $(3 + \sqrt{23}) - \sqrt{23}$

Solve $3 + \cancel{\sqrt{23}} - \cancel{\sqrt{23}}$

3 = Rational Number

(ii) $\frac{2\sqrt{7}}{7\sqrt{7}}$

Solve $\frac{2\cancel{\sqrt{7}}}{7\cancel{\sqrt{7}}}$

$\frac{2}{7}$ = Rational Number

(iv) $\frac{1}{\sqrt{2}}$

Solve where, 1 = Rational Number

$\sqrt{2}$ = Irrational Number

$\therefore \frac{1}{\sqrt{2}}$ = Irrational Number

(v) 2π

Solve where, 2 = Rational Number

π = Irrational Number

$\therefore 2\pi$ = Irrational Number

2. Simplify each of the following expressions:

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$

Solve $\Rightarrow 3(2 + \sqrt{2}) + \sqrt{3}(2 + \sqrt{2})$

$\Rightarrow 6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii) $(3+\sqrt{3})(3-\sqrt{3})$

Solve $3(3-\sqrt{3})+\sqrt{3}(3-\sqrt{3})$

$\Rightarrow 9 - \cancel{3\sqrt{3}} + \cancel{3\sqrt{3}} - \sqrt{9}$

$\Rightarrow 9 - \sqrt{9}$

$\Rightarrow 9 - \sqrt{3 \times 3}$

$\Rightarrow 9 - \sqrt{(3)^2}$

$\Rightarrow 9 - 3$

$\Rightarrow 6$

Using identity $(a+b)(a-b) = a^2 - b^2$

$\Rightarrow (3+\sqrt{3})(3-\sqrt{3})$

$\Rightarrow (3)^2 - \sqrt{(3)^2}$

$\Rightarrow 3^2 - 3$

$\Rightarrow 9 - 3$

$\Rightarrow 6$

(iii) $(\sqrt{5}+\sqrt{2})^2$

Solve Using identity $(a+b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow (\sqrt{5}+\sqrt{2})^2$

$\Rightarrow 5 + 2 + 2\sqrt{2}\sqrt{5}$

$\Rightarrow 7 + 2\sqrt{10}$

(iv) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

Solve Using identity $(a+b)(a-b) = a^2 - b^2$

$\Rightarrow (\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$\Rightarrow \sqrt{(5)^2} - \sqrt{(2)^2}$

$\Rightarrow 5 - 2$

$\Rightarrow 3$

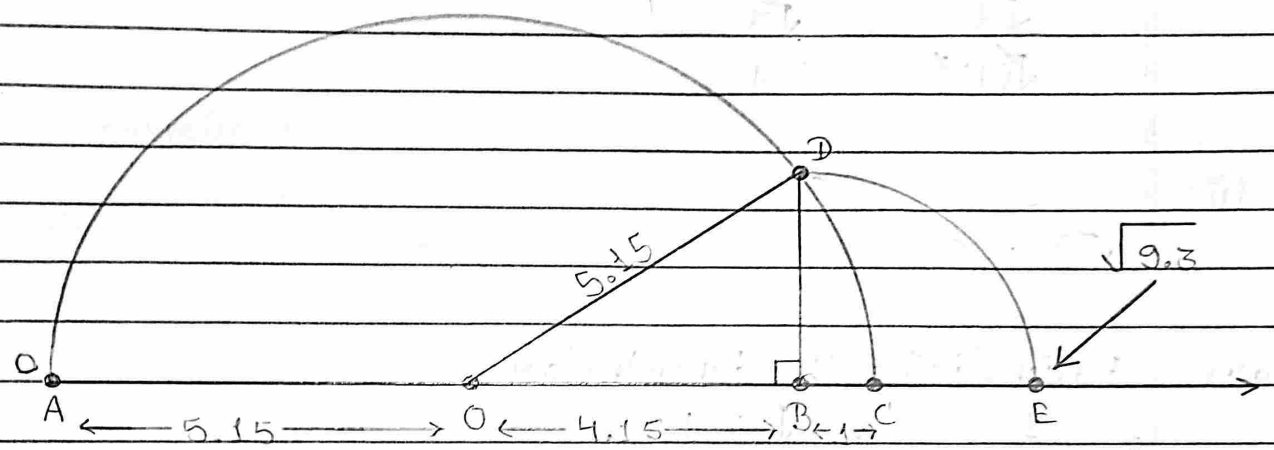
3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is $\pi = \frac{c}{d}$

This seems to contradict the fast that π is irrational. How will you resolve this contradiction?

Solve There is no contradiction. Remember that when you measure a length with a scale or any other device, you only get an approximate rational value. So, you may not realise that either c or d is irrational.

4. Represent $\sqrt{9.3}$ on the number line.

Solve



Draw a line of any length mark a distance 9.3 unit from a fixed point A. on a line to obtain a point B such that AB = 9.3. From B, mark a distance of one unit and mark the new point as C. We find the mid point of AC and mark that point as O. Draw a semicircle with centre O and Radius OA. Draw a line perpendicular to AC. Passing through B and Intersecting the semicircle at D. Then BD $= \sqrt{9.3}$.

In $\triangle OBD$, By using Pythagoras theorem,

$(OB)^2 = (BD)^2 + (OB)^2$

$(5.15)^2 = (BD)^2 + (4.15)^2$

$(BD)^2 = (5.15)^2 - (4.15)^2$

$(BD)^2 = (5.15 + 4.15)(5.15 - 4.15)$

$(BD)^2 = (9.30) \times 1$

$(BD)^2 = 9.3$

$BD = \sqrt{9.3}$

5. Rationalise the denominators of the following:

(i) $\frac{1}{\sqrt{7}}$

Solve Rationalising the denominator

$= \frac{1}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$

$= \frac{\sqrt{7}}{\cancel{\sqrt{}}(7)^{\cancel{2}}} = \frac{\sqrt{7}}{7}$

(ii) $\frac{1}{\sqrt{7}-2}$

Solve Rationalising the denominator

$= \frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$ $(a-b)(a+b) = a^2 - b^2$

$= \frac{\sqrt{7}+2}{\cancel{\sqrt{}}(7)^{\cancel{2}} - (2)^2}$

$= \frac{\sqrt{7}+2}{7-4}$

$= \frac{\sqrt{7}+2}{3}$

(iii) $\frac{1}{\sqrt{5}+\sqrt{2}}$

Solve Rationalising the denominator

$= \frac{1}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ $(a+b)(a-b) = a^2 - b^2$

$$= \frac{\sqrt{5}-\sqrt{2}}{\sqrt{(5)^2}-\sqrt{(2)^2}}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{3}$$

(iv) $\frac{1}{\sqrt{7}-\sqrt{6}}$

Solve Rationalising the denominator

$$= \frac{1}{\sqrt{7}-\sqrt{6}} \times \frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}+\sqrt{6}} \quad (a-b)(a+b) = a^2 - b^2$$

$$= \frac{\sqrt{7}+\sqrt{6}}{\sqrt{(7)^2}-\sqrt{(6)^2}}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{7-6}$$

$$= \frac{\sqrt{7}+\sqrt{6}}{1}$$

$$= \sqrt{7}+\sqrt{6}$$

Ex. 1.5

Formulas :-

1. $x^0 = 1$
2. $x^{-m} = \frac{1}{x^m} = \left(\frac{1}{x}\right)^m$
3. $(x^m)^n = x^{m \times n}$
4. $x^m \times x^n = x^{m+n}$
5. $x^m \div x^n = x^{m-n}$
6. $x^m \times y^m = (xy)^m$
7. $x^m \div y^m = \left(\frac{x}{y}\right)^m$
8. $\sqrt{8}\ \sqrt[n]{x} = (x)^{\frac{1}{n}}$
9. $\sqrt{x} = (x)^{\frac{1}{2}}$

1. Find :

(i) $64^{\frac{1}{2}}$

Solve $= 64^{\frac{1}{2}}$
$= (8 \times 8)^{\frac{1}{2}}$
$= (8^2)^{\frac{1}{2}}$ $\left[(x^m)^n = x^{m \times n}\right]$
$= 8^{\cancel{2} \times \frac{1}{\cancel{2}}}$
$= 8$

(ii) $32^{\frac{1}{5}}$

Solve $= 32^{\frac{1}{5}}$
$= (2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{5}}$
$= (2^5)^{\frac{1}{5}}$ $\left[(x^m)^n = x^{m \times n}\right]$
$= 2^{\cancel{5} \times \frac{1}{\cancel{5}}}$
$= 2$

(iii) $125^{\frac{1}{3}}$

Solve $= 125^{\frac{1}{3}}$
$= (5 \times 5 \times 5)^{\frac{1}{3}}$
$= (5^3)^{\frac{1}{3}}$ $\left[(x^m)^n = x^{m \times n}\right]$
$= 5^{\cancel{3} \times \frac{1}{\cancel{3}}}$
$= 5$

2. Find:

(i) $9^{\frac{3}{2}}$

Solve $= 9^{\frac{3}{2}}$

$= (3\times3)^{\frac{3}{2}}$

$= (3^2)^{\frac{3}{2}}$ $\boxed{(x^m)^n = x^{m\times n}}$

$= 3^{\cancel{2}\times\frac{3}{\cancel{2}}}$

$= 3^3$

$= 27$

(ii) $32^{\frac{2}{5}}$

Solve $= 32^{\frac{2}{5}}$

$= (2\times2\times2\times2\times2)^{\frac{2}{5}}$

$= (2^5)^{\frac{2}{5}}$ $\boxed{(x^m)^n = x^{m\times n}}$

$= 2^{\cancel{5}\times\frac{2}{\cancel{5}}}$

$= 2^2$

$= 4$

(iii) $16^{\frac{3}{4}}$

Solve $= 16^{\frac{3}{4}}$

$= \cancel{(4\times4)^{\frac{3}{4}}}$ $(2\times2\times2\times2)^{\frac{3}{4}}$

$= \cancel{(4^2)^{\frac{3}{4}}}$ $(2^4)^{\frac{3}{4}}$ $\boxed{(x^m)^n = x^{m\times n}}$

$= 4^{\cancel{2}\times\frac{3}{\cancel{4}\,2}}$ & $2^{4\times\frac{3}{4}}$

$= 4^{\frac{3}{2}}$ 2^3

$= 8$

(iv) $125^{-\frac{1}{3}}$

Solve $= 125^{-\frac{1}{3}}$

$= (5\times5\times5)^{-\frac{1}{3}}$

$= (5^3)^{-\frac{1}{3}}$ $\boxed{(x^m)^n = x^{m\times n}}$

$= 5^{\cancel{3}\cdot-\frac{1}{\cancel{3}}}$

$= 5^{-1}$ $\boxed{x^{-m} = \frac{1}{x^m} = \left(\frac{1}{x}\right)^m}$

$= \frac{1}{5^1}$

$= \left(\frac{1}{5}\right)^1 = \frac{1}{5}$

3. Simplify:

(i) $2^{\frac{2}{3}} \cdot 2^{\frac{1}{5}}$

Solve $= 2^{\frac{2}{3}} \times 2^{\frac{1}{5}}$ $\boxed{x^m \times x^n = x^{m+n}}$

$2^{\frac{2}{3}+\frac{1}{5}}$

$= 2^{\frac{13}{15}}$

(ii) $\left(\frac{1}{3^3}\right)^7$

Solve $= (3^{-3})^7$ $\boxed{\frac{1}{x^m} = x^{-m}}$

$= 3^{-3\times7}$

$= 3^{-21}$

(iii) $\frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$

Solve $= 11^{\frac{1}{2}-\frac{1}{4}}$ $\boxed{x^m \div x^n = x^{m-n}}$

$= 11^{\frac{1}{4}}$

(iv) $7^{\frac{1}{2}} \cdot 8^{\frac{1}{2}}$

Solve $7^{\frac{1}{2}} \times 8^{\frac{1}{2}}$ $\boxed{x^m \times y^m = (xy)^m}$

$(7\times8)^{\frac{1}{2}}$

$56^{\frac{1}{2}}$

09/07/25

V. Good