Chapter ⇒ 2

Polynomials

Ex. 2.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$ (ii) $y^2 + \sqrt{2}$

Solve Variable ⇒ x
Degree of variable ⇒ 2, 1.
∴ It is a polynomial in one variable

Solve Variable ⇒ y
Degree of variable = 2.
∴ It is a polynomial in one variable

(iii) $3\sqrt{t} + t\sqrt{2}$ (iv) $\cancel{\sqrt{2}}\ y + \frac{2}{y}$

Solve $3\sqrt{t} + t\sqrt{2} = 3t^{\frac{1}{2}} + t\sqrt{2}$
Variable ⇒ t
Degree of variable ⇒ $\frac{1}{2}$, 1.
∴ It is not a polynomial in one variable

Solve $y + \frac{2}{y} = y^1 + 2y^{-1}$
variable ⇒ y
Degree of variable = 1, −1.
∴ It is not a polynomial in one variable

(v) $x^{10} + y^3 t^{50}$

Solve variable ⇒ x, y, t.
Degree of variable ⇒ 10, 3, 50.
∴ It is not a polynomial in one variable.

2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$ (ii) $2 - x^2 + x^3$

Solve coefficients of $x^2 = 1$.

Solve coefficients of $x^2 = -1$

(iii) $\frac{\pi}{2}x^2 + x$

(iv) $\sqrt{2}x - 1$

Solve coefficient of $x^2 = 0$

Solve coefficient of $x^2 = \frac{\pi}{2}$

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solve One example of binomial of degree 35 is $2x^{35} - y$.

One example of monomial of degree 100 is $4z^{100}$.

4. Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$

Solve Term with the highest power of $x = 5x^3$
Exponent of x in this term $= 3$
$\therefore$ Degree of this polynomial $= 3$.

(ii) $4 - y^2$

Solve Term with the highest power of $y = -y^2$
Exponent of y in this term $= 2$
$\therefore$ Degree of this polynomial $= 2$.

(iii) $5t - \sqrt{7}$

Solve Term with the highest power of $t = 5t$
Exponent of t in this term $= 1$
$\therefore$ Degree of this polynomial $= 1$

(iv) 3

Solve It is a non-zero constant. So the degree of this polynomial is zero.

5. Classify the following as linear, quadratic and cubic polynomials

(i) $x^2 + x$ (ii) $x - x^3$

Solve Degree $\Rightarrow 2$
$\therefore$ It is a quadratic polynomial

Solve Degree $\Rightarrow 3$
$\therefore$ It is a cubic polynomial

(iii) $y + y^2 + 4$ (iv) $1 + x$

Solve Degree $\Rightarrow 2$
$\therefore$ It is a quadratic polynomial

Solve Degree $\Rightarrow 1$
$\therefore$ It is a linear polynomial

(v) $3t$ (vi) r^2

Solve Degree $\Rightarrow 1$
$\therefore$ It is a ~~quadratic~~ linear polynomial

Solve Degree $\Rightarrow 2$
$\therefore$ It is a quadratic polynomial

(vii) $7x^3$

Solve Degree $\Rightarrow 3$
$\therefore$ It is a cubic polynomial

Ex. 2.2

1. Find the value of the polynomial $5x - 4x^2 + 3$ at

(i) $x = 0$ (ii) $x = -1$ (iii) $x = 2$

Solve (i) Let $p(x) = 5x - 4x^2 + 3$

if $x = 0$ Then

$p(0) = 5 \times 0 - 4 \times 0^2 + 3$

$= 0 - 4 \times 0 + 3$

$= 0 - 0 + 3$

$p(0) = 3$

(ii) if $x = -1$ Then

$p(-1) = 5 \times (-1) - 4(-1)^2 + 3$

$= -5 - 4 \times 1 + 3$

$= -5 - 4 + 3$

$= -9 + 3$

$p(-1) = -6$

(iii) if $x = 2$ Then

$p(2) = 5 \times 2 - 4 \times 2^2 + 3$

$= 10 - 4 \times 4 + 3$

$= 10 - 16 + 3$

$= 13 - 16$

$p(2) = -3$

2. Find $p(0)$, $p(1)$ and $p(2)$ for each of the following polynomials:

(i) $p(y) = y^2 - y + 1$

Solve $p(y) = y^2 - y + 1$

if $y = 0$ Then

$p(0) = 0^2 - 0 + 1$

$\boxed{p(0) = 1}$

if $y = 1$ Then

$p(1) = 1^2 - 1 + 1$

$p(1) = 1 - \cancel{1} + \cancel{1}$

$\boxed{p(1) = 1}$

if $y = 2$ Then

$p(2) = 2^2 - 2 + 1$

$p(2) = 4 - 2 + 1$

$p(2) = 5 - 2$

$\boxed{p(2) = 3}$

(ii) $p(t) = 2 + t + 2t^2 - t^3$

Solve: $p(t) = 2 + t + 2t^2 - t^3$

if $t = 0$ Then

$p(0) = 2 + 0 + 2 \times 0^2 - 0^3$

$p(0) = 2 + 0 + 0 - 0$

$\boxed{p(0) = 2}$

if $t = 1$ Then

$p(1) = 2 + 1 + 2 \times 1^2 - 1^3$

$p(1) = 2 + 1 + 2 - 1$

$p(1) = 5 - 1$

$\boxed{p(1) = 4}$

if $t = 2$ Then

$p(2) = 2 + 2 + 2 \times 2^2 (-2)^3$

$p(2) = 2 + 2 + 8 - 8$

$p(2) = 12 - 8$

$\boxed{p(2) = 4}$

(iii) $p(x) = x^3$

Solve $p(x) = x^3$

if $x = 0$ Then

$p(0) = 0^3$

$\boxed{p(0) = 0}$

if $x = 1$ Then

$p(1) = 1^3$

$\boxed{p(1) = 1}$

if $x = 2$ Then

$p(2) = 2^3$

$\boxed{p(2) = 8}$

(iv) $p(x) = (x-1)(x+1)$

Solve $p(x) = (x-1)(x+1)$

if $x = 0$ Then

$p(0) = (0-1)(0+1)$

$p(0) = -1 \times 1$

$\boxed{p(0) = -1}$

if $x = 1$ Then

$p(1) = (1-1)(1+1)$

$p(1) = 0 \times 2$

$\boxed{p(1) = 0}$

if $x = 2$ Then

$p(2) = (2-1)(2+1)$

$p(2) = 1 \times 3$

$\boxed{p(2) = 3}$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x) = 3x + 1, \; x = -\frac{1}{3}$

Solve if $x = -\frac{1}{3}$ Then

$p\left(-\frac{1}{3}\right) = \cancel{3} \times \left(-\frac{1}{\cancel{3}}\right) + 1$

$p\left(-\frac{1}{3}\right) = -1 + 1$

$p\left(-\frac{1}{3}\right) = 0$

yes, $-\frac{1}{3}$ is a zeroes of $p(x)$.

(ii) $p(x) = 5x - \pi, \; x = \frac{4}{5}$

Solve if $x = \frac{4}{5}$ Then

$p\left(\frac{4}{5}\right) = \cancel{5} \times \frac{4}{\cancel{5}} - \pi$

$p\left(\frac{4}{5}\right) = \boxed{4 - \pi} \neq 0$

No, $\frac{4}{5}$ is not a zeroes of $p(x)$

(iii) $p(x) = x^2 - 1, \; x = 1, -1$

Solve if $x = 1$ Then

$p(1) = 1^2 - 1$

$p(1) = 1 - 1$

$p(1) = 0$

yes, 1 is a zeroes of $p(x)$.

if $x = -1$ Then

$p(-1) = (-1)^2 - 1$

$p(-1) = 1 - 1$

$p(-1) = 0$

yes, -1 is a zeroes of $p(x)$.

(iv) $p(x) = (x+1)(x-2), \; x = -1, 2.$

Solve if $x = -1$ Then

$p(-1) = (-1+1)(-1-2)$

$p(-1) = 0 \times (-3)$

$p(-1) = 0$
yes, -1 is a zeroes of (px).

if $x = 2$ Then
$p(2) = (2+1)(2-2)$
$p(2) = 3 \times 0$
$p(2) = 0$
yes, 2 is a zeroes of $p(x)$

(v) $p(x) = x^2, x = 0$

Solve if $x = 0$ Then
$p(0) = 0^2$
$p(0) = 0$
yes, 0 is a zeroes of $p(x)$.

(vi) $p(x) = lx + m, x = -\frac{m}{l}$

Solve if $x = -\frac{m}{l}$ Then
$p\left(-\frac{m}{l}\right) = l \times \left(-\frac{m}{l}\right) + m$
$p\left(-\frac{m}{l}\right) = -m + m$
$p\left(-\frac{m}{l}\right) = 0$
yes, $-\frac{m}{l}$ is a zeroes of $p(x)$.

(vii) $p(x) = 3x^2 - 1, x = -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

Solve if $x = -\frac{1}{\sqrt{3}}$ Then
$p\left(-\frac{1}{\sqrt{3}}\right) = 3 \times \left(-\frac{1}{\sqrt{3}}\right)^2 - 1$
$p\left(-\frac{1}{\sqrt{3}}\right) = \cancel{3} \times \frac{1}{\cancel{3}} - 1$
$p\left(-\frac{1}{3}\right) = 1 - 1$
$p\left(-\frac{1}{3}\right) = 0$
yes, $-\frac{1}{\sqrt{3}}$ is a zeroes of $p(x)$

if $x = \frac{2}{\sqrt{3}}$ Then
$p\left(\frac{2}{\sqrt{3}}\right) = 3 \times \left(\frac{2}{\sqrt{3}}\right)^2 - 1$
$p\left(\frac{2}{\sqrt{3}}\right) = \cancel{3} \times \frac{4}{\cancel{3}} - 1$
$p\left(\frac{2}{3}\right) = 4 - 1 = \boxed{3} \neq 0$
No, $\frac{2}{\sqrt{3}}$ is not a zeroes of $p(x)$.

(viii) $p(x) = 2x + 1, x = \frac{1}{2}$

solve if $x = \frac{1}{2}$ Then

$p\left(\frac{1}{2}\right) = \cancel{2} \times \frac{1}{\cancel{2}} + 1$

$p\left(\frac{1}{2}\right) = 1 + 1$

$p\left(\frac{1}{2}\right) = \boxed{2} \neq 0$

No, $\frac{1}{2}$ is not a zeroes of $p(x)$.

4. Find the zero of the polynomial in each of the following cases

(i) $p(x) = x + 5$

solve For finding zero

$p(x) = 0$

$x + 5 = 0$

$x = 0 - 5$

$\boxed{x = -5}$

hence, -5 is a zero of the polynomial $p(x)$.

(ii) $p(x) = x - 5$

solve For finding zero

$p(x) = 0$

$x - 5 = 0$

$x = 0+5$

$\boxed{x = 5}$

hence, 5 is a zero of the polynomial $p(x)$.

(iii) $p(x) = 2x+5$

Solve For finding zero

$p(x) = 0$

$2x+5 = 0$

$2x = 0-5$

$2x = -5$

$\boxed{x = \frac{-5}{2}}$

hence, $\frac{-5}{2}$ is a zero of the polynomial $p(x)$.

(iv) $p(x) = 3x-2$

Solve For finding zero

$p(x) = 0$

$3x-2 = 0$

$3x = 0+2$

$3x = 2$

$\boxed{x = \frac{2}{3}}$

hence, $\frac{2}{3}$ is a zero of the polynomial $p(x)$.

(v) $p(x) = 3x$

Solve For finding zero

$p(x) = 0$

$3x = 0$

$\boxed{x = \frac{0}{3}}$

hence, 0 is a zero of the polynomial $p(x)$.

(vi) $p(x) = ax$, $a \neq 0$

Solve For finding zero

$p(x) = 0$

$ax = 0$

$\boxed{x = \frac{0}{a}}$

hence, 0 is a zero of the polynomial $p(x)$.

(vii) $p(x) = cx + d$, $c \neq 0$, c, d are real numbers.

Solve For finding zero

$p(x) = 0$

$cx + d = 0$

$cx = 0 - d$

$cx = -d$

$\boxed{x = \frac{-d}{c}}$

hence, $\frac{-d}{c}$ is a zero of the polynomial $p(x)$.

Ex. 2.3

1. Determine which of the following polynomials has $(x+1)$ a factor:

(i) $x^3 + x^2 + x + 1$

Solve Given that,

$p(x) = x^3 + x^2 + x + 1$

and divisor $\Rightarrow x + 1 = 0 \Rightarrow x = -1$

By remainder theorem,

$$\text{Remainder} = p(-1)$$
$$= (-1)^3 + (-1)^2 + (-1) + 1$$
$$= \cancel{-1} + \cancel{1} - \cancel{1} + \cancel{1}$$
$$\text{Remainder} = 0$$

$\therefore$ yes, $(x+1)$ is a factor of $p(x)$.

(ii) $x^4 + x^3 + x^2 + x + 1$

Solve Given that,

$p(x) = x^4 + x^3 + x^2 + x + 1$

and divisor $\Rightarrow x + 1 = 0 \Rightarrow x = -1$

By remainder theorem,

$$\text{Remainder} = p(-1)$$
$$= (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$
$$= \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + 1$$
$$\text{Remainder} = 1$$

$\therefore$ No, $(x+1)$ is not a factor of $p(x)$.

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Solve Given that,

$p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

and divisor $\Rightarrow x + 1 = 0 \Rightarrow x = -1$

By Remainder theorem,

Remainder $= p(-1)$

$= (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - \cancel{3} + \cancel{3} - \cancel{1} + \cancel{1}$

Remainder $= 1$

$\therefore$ No, $(x+1)$ is not a factor of $p(x)$.

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Solve Given that,

$p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

and divisor $\Rightarrow x + 1 = 0 \Rightarrow x = -1$

By Remainder theorem,

Remainder $= p(-1)$

$= (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$

$= -\cancel{2} + \cancel{2} + 2\sqrt{2}$

Remainder $= 2\sqrt{2}$

where, Remainder $\neq 0$

$\therefore$ No, $(x+1)$ is not a factor of $p(x)$.

2. Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

Solve: Given that,

$$p(x) = 2x^3 + x^2 - 2x - 1$$
$$g(x) = x + 1$$

Let divisor $= x + 1 = 0 \Rightarrow x = -1$

By Remainder theorem,

$$\text{Remainder} = p(-1)$$
$$= 2(-1)^3 + (-1)^2 - 2(-1) - 1$$
$$= -\cancel{2} + \cancel{1} + \cancel{2} - \cancel{1}$$
$$\text{Remainder} = 0$$

$\therefore$ yes, $g(x)$ is a factor of $p(x)$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

Solve: Given that,

$$p(x) = x^3 + 3x^2 + 3x + 1$$
$$g(x) = x + 2$$

Let divisor $= x + 2 = 0 \Rightarrow x = -2$

By Remainder theorem,

$$\text{Remainder} = p(-2)$$
$$= (-2)^3 + 3(-2)^2 + 3(-2) + 1$$
$$= -8 + 12 - 6 + 1$$
$$= -14 + 13$$
$$\text{Remainder} = -1$$

where, Remainder $\neq 0$

$\therefore$ No, $g(x)$ is not a factor of $p(x)$.

(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Solve Given that,

$p(x) = x^3 - 4x^2 + x + 6$

$g(x) = x - 3$

Let divisor $= x - 3 = 0 \Rightarrow x = \cancel{4}3$

By Remainder theorem,

$$\text{Remainder} = p(3)$$
$$= (3)^3 - 4(3)^2 + 3 + 6$$
$$= \cancel{9}\ 27 - 36 + 3 + 6$$
$$= 36 - 36$$
$$\text{Remainder} = 0$$

$\therefore$ yes, $g(x)$ is a factor of $p(x)$.

3. Find the value of k, if $x-1$ is a factor of $p(x)$ in each of the following cases :

(i) $p(x) = x^2 + x + k$

Solve If $x - 1$ is a factor of $p(x)$, then $p(1) = 0$

$\therefore$ By factor theorem,

$$p(1) = 0$$
$$(1)^2 + 1 + k = 0$$
$$1 + 1 + k = 0$$
$$2 + k = 0$$
$$\boxed{k = -2}$$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

Solve If $x-1$ is a factor of $p(x)$, then $p(1) = 0$

$\therefore$ By factor theorem,

$$p(1) = 0$$

$$2(1)^2 + k \times 1 + \sqrt{2} = 0$$

$$2 + k + \sqrt{2} = 0$$

$$\boxed{k = -(2 + \sqrt{2})}$$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

Solve If $x-1$ is a factor of $p(x)$, then $p(1) = 0$

$\therefore$ By factor theorem,

$$p(1) = 0$$

$$k(1)^2 - \sqrt{2} \times 1 + 1 = 0$$

$$k \times 1 - \sqrt{2} + 1 = 0$$

$$k - \sqrt{2} + 1 = 0$$

$$\boxed{k = \sqrt{2} - 1}$$

(iv) $p(x) = kx^2 - 3x + k$

solve If $x-1$ is a factor of $p(x)$, then $p(1) = 0$

$\therefore$ By factor theorem,

$$p(1) = 0$$

$$k(1)^2 - 3 \times 1 + k = 0$$

$$k \times 1 - 3 + k = 0$$

$$k - 3 + k = 0$$

$$2k = 3$$

$$\boxed{k = \frac{3}{2}}$$

4. Factorise :

(i) $12x^2 - 7x + 1$

Solve $= 12x^2 - 4x - 3x + 1$

$= 4x(3x-1) - 1(3x-1)$

$= (3x-1)(4x-1)$

(ii) $2x^2 + 7x + 3$

Solve $= 2x^2 + 6x + x + 3$

$= 2x(x+3) + 1(x+3)$

$= (x+3)(2x+1)$

(iii) $6x^2 + 5x - 6$

Solve $= 6x^2 + 9x - 4x - 6$

$= 3x(2x+3) - 2(2x+3)$

$= (2x+3)(3x-2)$

(iv) $3x^2 - x - 4$

Solve $= 3x^2 + 3x - 4x - 4$

$= 3x(x+1) - 4(x+1)$

$= (x+1)(3x-4)$

5. Factorise :

(i) $x^3 - 2x^2 - x + 2$

(ii) $x^3 - 3x^2 - 9x - 5$

Solve $= x^3 - x - 2x^2 + 2$

$= x(x^2-1) - 2(x^2-1)$

$= (x^2-1)(x-2)$

$= (x+1)(x-1)(x-2)$

Ye 5th question galat hai next page per dekhen sahi 5th question ko .

Thank you 👍

5. (i) Factorise :   Sahi 5th question

(i) $x^3 - 2x^2 - x + 2$

Solve Let $p(x) = x^3 - 2x^2 - x + 2$

We shall look the factors of 2 are ± 1, ± 2

By trial,

$$p(1) = (1)^3 - 2(1)^2 - 1 + 2$$
$$= 1 - 2 \times 1 - 1 + 2$$
$$= \cancel{1} - 2 - \cancel{1} + \cancel{2}$$
$$p(1) = 0$$

So, $(x-1)$ is a factor of $p(x)$

Now,

$$x^3 - 2x^2 - x + 2 = x^2(x-1) - x(x-1) - 2(x-1)$$
$$= (x-1)[x^2 - x - 2]$$
$$= (x-1)[x^2 + x - 2x - 2]$$
$$= (x-1)[x(x+1) - 2(x+1)]$$
$$= (x-1)(x+1)(x-2)$$

(ii) $x^3 - 3x^2 - 9x - 5$

Solve Let $p(x) = x^3 - 3x^2 - 9x - 5$

We shall look the factors of -5 are ± 1, ± 5

By trial,

$$p(-1) = (-1)^3 - 3(-1)^2 - 9(-1) - 5$$
$$= -1 - 3 \times 1 + 9 - 5$$
$$= -1 - 3 + 9 - 5$$
$$= -9 + 9$$
$$p(-1) = 0$$

So, $(x+1)$ is a factor of $p(x)$

Now,

$$x^3-3x^2-9x-5 = x^2(x+1)-4x(x+1)-5(x+1)$$
$$= (x+1)[x^2-4x-5]$$
$$= (x+1)[x^2+x-5x-5]$$
$$= (x+1)[x(x+1)-5(x+1)]$$
$$= (x+1)(x+1)(x-5)$$

(iii) $x^3+13x^2+32x+20$

Solve Let $p(x) = x^3+13x^2+32x+20$

We shall look the factors of 20 are $\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

By trial,

$$p(-1) = (-1)^3+13(-1)^2+32(-1)+20$$
$$= -1+13\times 1+32\times(-1)+20$$
$$= -1+13-32+20$$
$$= -33+33$$
$$p(-1) = 0$$

So, $(x+1)$ is a factor of $p(x)$

Now,

$$x^3+13x^2+32x+20 = x^2(x+1)+12x(x+1)+20(x+1)$$
$$= (x+1)[x^2+12x+20]$$
$$= (x+1)[x^2+2x+10x+20]$$
$$= (x+1)[x(x+2)+10(x+2)]$$
$$= (x+1)(x+2)(x+10)$$

(iv) $2y^3 + y^2 - 2y - 1$

Solve Let $p(y) = 2y^3 + y^2 - 2y - 1$

We shall look the factors of -1 are ± 1, ± 2

By trial,

$$p(1) = 2(1)^3 + (1)^2 - 2(1) - 1$$
$$= 2 \times 1 + 1 - 2 - 1$$
$$= \cancel{2} + \cancel{1} - \cancel{2} - \cancel{1}$$
$$p(1) = 0$$

So, $(y-1)$ is a factor of $p(y)$

Now,

$$2y^3 + y^2 - 2y - 1 = 2y^2(y-1) + 3y(y-1) + 1(y-1)$$
$$= (y-1)[2y^2 + 3y + 1]$$
$$= (y-1)[2y^2 + 2y + y + 1]$$
$$= (y-1)[2y(y+1) + 1(y+1)]$$
$$= (y-1)(y+1)(2y+1)$$

Ex. 2.4

Formulas :-

(i) $(x+y)^2 = x^2 + y^2 + 2xy$

(ii) $(x-y)^2 = x^2 + y^2 - 2xy$

(iii) $x^2 - y^2 = (x+y)(x-y)$

(iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(x-a)(x+b) = x^2 - (a-b)x - ab$

(vi) $(x-a)(x-b) = x^2 - (a+b)x + ab$

(vii) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(viii) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(ix) $(x-y)^3 = x^3 - y^3 - 3xy(x-y) = x^3 - 3x^2y + 3xy^2 - y^3$

(x) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

(xi) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

(xii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

(xiii) $(x+a)(x-b) = x^2 + (a-b)x - ab$

1. Use suitable identities to find the following product :

(i) $(x+4)(x+10)$

Solve By using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$

$= x^2 + (4+10)x + 4 \times 10$

$= x^2 + 14x + 40$

(ii) $(x+8)(x-10)$

Solve By using identity $(x+a)(x-b) = x^2 + (a-b)x - ab$

$= x^2 + (8-10)x - 8 \times 10$

$= x^2 - 2x - 80$

(iii) $(3x+4)(3x-5)$

Solve By using identity $(x+a)(x-b) = x^2 + (a-b)x - ab$

$= (3x)^2 + (4-5)3x - 4 \times 5$

$= 9x^2 - 3x - 20$

(iv) $\left(y^2 + \frac{3}{2}\right)\left(y^2 - \frac{3}{2}\right)$

Solve By using identity $(x+y)(x-y) = x^2 - y^2$

$= (y^2)^2 - \left(\frac{3}{2}\right)^2$

$= y^4 - \frac{9}{4}$

(v) $(3-2x)(3+2x)$

Solve By using identity $(x-y)(x+y) = x^2 - y^2$

$= (3)^2 - (2x)^2$

$= 9 - 4x^2$

2. Evaluate the fallowing products without multiplying directly:

(i) 103×107

Solve $103 \times 107 = (100+3)(100+7)$

By using identity $(x+a)(x+b) = x^2 + (a+b)x + ab$

$= (100)^2 + (3+7)100 + 3 \times 7$

$= 10000 + 1000 + 21$

$= 11021$

(ii) 95×96

Solve $95 \times 96 = (100-5)(100-4)$

By using identity $(x-a)(x-b) = x^2 - (a+b)x + ab$

$= (100)^2 - (5+4)100 + 5 \times 4$

$= 10000 - 900 + 20$

$= 9100 + 20$

$= 9120$

(iii) 104×96

Solve $104 \times 96 = (100+4)(100-4)$

By using identity $(x+y)(x-y) = x^2 - y^2$

$= (100)^2 - (4)^2$

$= 10000 - 16$

$= 9984$

3. Factorise the following using appropriate identities:

(i) $9x^2 + 6xy + y^2$

Solve $= (3x)^2 + 2 \times 3x \times y + (y)^2$

By using identity $x^2 + 2xy + y^2 = \cancel{x^2 + y^2}\ (x+y)^2$

$= (3x+y)^2$

$= (3x+y)(3x+y)$

(ii) $4y^2 - 4y + 1$

Solve $= (2y)^2 - 2 \times 2y \times 1 + (1)^2$

By using identity $x^2 - 2xy + y^2 = (x-y)^2$

$= (2y-1)^2$

$= (2y-1)(2y-1)$

(iii) $x^2 - \frac{y^2}{100}$

Solve $= x^2 - \left(\frac{y}{10}\right)^2$

By using identity $x^2 - y^2 = (x+y)(x-y)$

$= \left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$

4. Expand each of the following, usingsuitable identities:

(i) $(x + 2y + 4z)^2$

Solve By using identity $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

$= (x)^2 + (2y)^2 + (4z)^2 + 2 \times x \times (2y) + 2 \times (2y) \times (4z) + 2 \times (4z) \times x$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$

(ii) $(2x-y+z)^2$

Solve $(2x-y+z)^2 = (2x+(-y)+z)^2$

By using identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$= (2x)^2+(-y)^2+(z)^2+2\times(2x)\times(-y)+2\times(-y)\times(z)+2\times(z)\times(2x)$

$= 4x^2+y^2+z^2+4xy-2yz+4zx$

(iii) $(-2x+3y+2z)^2$

Solve By using identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$= (-2x)^2+(3y)^2+(2z)^2+2\times(-2x)\times 3y+2\times 3y\times 2z+2\times 2z\times(-2x)$

$= 4x^2+9y^2+4z^2-12xy+12yz-8xz$

(iv) $(3a-7b-c)^2$

Solve $(3a-7b-c)^2 = [3a+(-7b)+(-c)]^2$

By using identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$= (3a)^2+(-7b)^2+(-c)^2+2\times 3a\times(-7b)+2\times(-7b)\times(-c)+2\times(-c)\times 3a$

$= 9a^2+49b^2+c^2-42ab+14bc-6ac$

(v) $(-2x+5y-3z)^2$

Solve $(-2x+5y-3z)^2 = [-2x+5y+(-3z)]^2$

By using identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$= (-2x)^2+(5y)^2+(-3z)^2+2\times(-2x)\times 5y+2\times 5y\times(-3z)+2\times(-3z)\times(-2x)$

$= 4x^2+25y^2+9z^2-20xy-30yz+12zx$

(vi) $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2$

Solve $\left[\frac{1}{4}a - \frac{1}{2}b + 1\right]^2 = \left[\frac{1}{4}a + \left(-\frac{1}{2}b\right) + 1\right]^2$

By using identity $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$

$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + (1)^2 + \cancel{2}^1 \times \frac{1}{\cancel{4}_2}a \times \left(-\frac{1}{2}b\right) + \cancel{2} \times \left(-\frac{1}{\cancel{2}}b\right) \times 1 +$

${}^1\cancel{2} \times 1 \times \frac{1}{\cancel{4}_2}a$

$= \frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}$

5. Factorise :

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

Solve $= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times 2x$

By using identity $x^2+y^2+z^2+2xy+2yz+2zx = (x+y+z)^2$

$= (2x+3y-4z)^2$

$= (2x+3y-4z)(2x+3y-4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

Solve $= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + 2 \times (-\sqrt{2}x) \times y + 2 \times y \times (2\sqrt{2}z) +$
$2 \times (2\sqrt{2}z)(-\sqrt{2}x)$

By using identity $x^2+y^2+z^2+2xy+2yz+2zx = (x+y+z)^2$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

6. Write the following cubes in expanded form :

(i) $(2x+1)^3$

Solve By using identity $(x+y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$

$= (2x)^3 + (1)^3 + 3 \times (2x)^2 \times 1 + 3 \times 2x \times (1)^2$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a-3b)^3$

Solve By using identity $(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$

$= (2a)^3 - (3b)^3 - 3 \times (2a)^2 \times 3b + 3 \times 2a \times (3b)^2$

$= 8a^3 - 27b^3 - 36a^2b + \cancel{54ab^2}\ 54ab^2$

(iii) $\left[\frac{3}{2}x + 1\right]^3$

Solve By using identity $(x+y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$

$= \left(\frac{3}{2}x\right)^3 + (1)^3 + 3 \times \left(\frac{3}{2}x\right)^2 \times 1 + 3 \times \frac{3}{2}x \times (1)^2$

$= \frac{27x^3}{8} + 1 + \frac{27x^2}{2} + \frac{9}{2}x$

$= \frac{27}{8}x^3 + \frac{27}{2}x^2 + \frac{9}{2}x + 1$

(iv) $\left[x - \frac{2}{3}y\right]^3$

Solve By using identity $(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$

$= (x)^3 - \left(\frac{2}{3}y\right)^3 - 3 \times (x)^2 \times \frac{2}{3}y + 3 \times x \times \left(\frac{2}{3}y\right)^2$

$= x^3 - \frac{8y^3}{27} - \frac{\cancel{6}^{2}x^2y}{\cancel{3}_{1}} + \frac{\cancel{12}^{4}xy^2}{\cancel{9}_{3}}$

$= x^3 - \frac{8y^3}{27} - 2x^2y + \frac{4xy^2}{3}$

7. Evaluate the following using suitable identities:

(i) $(99)^3$

Solve $(99)^3 = (100-1)^3$

By using identity $(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$

$= (100)^3 - (1)^3 - 3 \times (100)^2 \times 1 + 3 \times 100 \times (1)^2$

$= 1000000 - 1 - 30000 + 300$

$= 1000300 - 30001$

$= 970299$

(ii) $(102)^3$

Solve $(102)^3 = (100+2)^3$

By using identity $(x+y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$

$= (100)^3 + (2)^3 + 3 \times (100)^2 \times 2 + 3 \times 100 \times (2)^2$

$= 1000000 + 8 + 60000 + 1200$

$= 1061208$

(iii) $(998)^3$

Solve $(998)^3 = (1000-2)^3$

By using identity $(x-y)^3 = x^3 - y^3 - 3x^2y + 3xy^2$

$= (1000)^3 - (2)^3 - 3 \times (1000)^2 \times 2 + 3 \times 1000 \times (2)^2$

$= 1000000000 - 8 - 6000000 + 12000$

$= 1000012000 - 6000008$

$= 994001992$

8. Factorise each of the following :

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

Solve $= (2a)^3 + (b)^3 + 3(2a)^2 b + 3(2a)b^2$

By using identity $x^3 + y^3 + 3x^2y + 3xy^2 = (x+y)^3$

$= (2a+b)^3$

$= (2a+b)(2a+b)(2a+b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$

Solve $= (2a)^3 - (b)^3 - 3 \times (2a)^2 \times b + 3 \times 2a \times (b)^2$

By using identity $x^3 - y^3 - 3x^2y + 3xy^2$

$= (2a - b)^3$

$= (2a-b)(2a-b)(2a-b)$

(iii) $27 - 125a^3 - 135a + 225a^2$

Solve $= (3)^3 - (5a)^3 - 3 \times (3)^2 \times 5a + 3 \times 3 \times (5a)^2$

By using identity $x^3 - y^3 - 3x^2y + 3xy^2$

$= (3 - 5a)^3$

$= (3-5a)(3-5a)(3-5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$

Solve $= (4a)^3 - (3b)^3 - 3 \times (4a)^2 \times 3b + 3 \times 4a \times (3b)^2$

By using identity $x^3 - y^3 - 3x^2y + 3xy^2$

$= (4a - 3b)^3$

$= (4a-3b)(4a-3b)(4a-3b)$

(v) $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

Solve $= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times (3p)^2 \times \left(\frac{1}{6}\right) + 3 \times 3p \times \left(\frac{1}{6}\right)^2$

By using identity $x^3 - y^3 - 3x^2y + 3xy^2$

$= \left(3p - \frac{1}{6}\right)^3$

$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$

9. Verify :

(i) $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

Solve Taking R.H.S.

$= (x+y)(x^2 - xy + y^2)$

$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$

$= x^3 - \cancel{x^2y} + \cancel{xy^2} + \cancel{x^2y} - \cancel{xy^2} + y^3$

$= x^3 + y^3$

$=$ L.H.S.

hence, it is verified.

(ii) $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Solve Taking R.H.S.

$= (x-y)(x^2 + xy + y^2)$

$= x(x^2 + xy + y^2) - y(x^2 + xy + y^2)$

$= x^3 + \cancel{x^2y} + \cancel{xy^2} - \cancel{x^2y} - \cancel{xy^2} - y^3$

$= x^3 - y^3$

$=$ L.H.S.

hence, it is verified.

10. Factorise each of the following :

(i) $27y^3 + 125z^3$

Solve $= (3y)^3 + (5z)^3$

By using identity $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

$= \cancel{(3y)^3}\,(3y + 5z)\,[(3y)^2 - 3y \times 5z + (5z)^2]$

$= (3y + 5z)\,[9y^2 - 15yz + 25z^2]$

(ii) $64m^3 - 343n^3$

Solve $= (4m)^3 - (7n)^3$

By using identity $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

$= (4m - 7n)\,[(4m)^2 + 4m \times 7n + (7n)^2]$

$= (4m - 7n)\,[16m^2 + 28mn + 49n^2]$

11. Factorise : $27x^3 + y^3 + z^3 - 9xyz$

Solve $(3x)^3 + y^3 + z^3 - 3 \times (3x) \times y \times z$

By using $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$= (3x+y+z)\,[(3x)^2 + y^2 + z^2 - 3x \times y - yz - z \times 3x]$

$= (3x+y+z)\,[9x^2 + y^2 + z^2 - 3xy - yz - 3zx]$

12. Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x+y+z)[(x-y)^2 + (y-z)^2 + (z-x)^2]$

Solve Taking R.H.S.

$= \frac{1}{2}(x+y+z)[(x-y)^2 + (y-z)^2 + (z-x)^2]$

$= \frac{1}{2}(x+y+z)[x^2+y^2-2xy+y^2+z^2-2yz+z^2+x^2-2zx$

$= \frac{1}{2}(x+y+z)[2x^2+2y^2+2z^2-2xy-2yz-2zx]$

$= \cancel{2} \times \frac{1}{\cancel{2}}(x+y+z)[x^2+y^2+z^2-xy-yz-zx]$

$= x^3+y^3+z^3-3xyz$

$=$ L.H.S.

hence, it is verified.

13. If $x+y+z=0$, Show that $x^3+y^3+z^3=3xyz$.

Solve We know that,

$x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

if $x+y+z=0$ Then

$= x^3+y^3+z^3-3xyz = 0[x^2+y^2+z^2-xy-yz-zx]$

$= x^3+y^3+z^3-3xyz=0$

$= x^3+y^3+z^3 = 3xyz$ Proved

14. Without actually calculating the cubes, find the value of each of the following:

(i) $(-12)^3+(7)^3+(5)^3$

Solve By using identity $x^3+y^3+z^3=(x+y+z)(x^2+y^2+z^2-xy-yz-zx)+3x$

$=(-12+7+5)[(-12)^2+(7)^2+(5)^2-(-12)\times 7-7\times 5-5\times(-12)]+3$

$\times(-12)\times 7\times 5$

$=(-\cancel{12}+\cancel{12})[(-12)^2+(7)^2+(5)^2-(-12)\times 7-7\times 5-5\times(-12)]-1260$

$=0[(-12)^2+(7)^2+(5)^2-(-12)\times 7-7\times 5-5\times(-12)]-1260$

$= -1260$

(ii) $(28)^3 + (-15)^3 + (-13)^3$

Solve By using identity $x^3+y^3+z^3 = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)+3xyz$

$= [28+(-15)+(-13)][(28)^2+(-15)^2+(-13)^2-28\times(-15)-(-15)\times(-13)-(-13)\times 28] + 3\times 28\times(-15)\times(-13)$

$= [28-15-13][(28)^2+(-15)^2+(-13)^2-28\times(-15)-(-15)\times(-13)-(-13)\times 28]16380$

$= [28-28][(28)^2+(-15)^2+(-13)^2-28\times(-15)-(-15)\times(-13)-(-13)\times 28]\ 16380$

$= 0\ [(28)^2+(-15)^2+(-13)^2-28\times(-15)-(-15)\times(-13)-(-13)\times 28]\ 16380$

$= 16380$

15. Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:

(i)	(ii)
Area : $25a^2 - 35 + 12$	Area : $35y^2 + 13y - 12$

Solve(i) Given that,

Area of Rectangle $= 25a^2 - 35a + 12$

$l \times b = 25a^2 - 20a - 15a + 12$

$= 5a(5a-4) - 3(5a-4)$

$l \times b = (5a-3)(5a-4)$

hence, length $= (5a-3)$, breadth $= (5a-4)$.

Solve (ii) Given that,

$$\text{Area of Rectangle} = 35y^2 + 13y - 12$$
$$l \times b = 35y^2 + 28y - 15y - 12$$
$$= 7y(5y+4) - 3(5y+4)$$
$$l \times b = (7y-3)(5y+4)$$

hence, length $= (7y-3)$, breadth $= (5y+4)$.

16. What are the possible expressions for the dimensions of the cuboids whose volumes are given below ?

(i)	(ii)
Volume : $3x^2 - 12x$	Volume : $12ky^2 + 8ky - 20k$

Solve (i) Given that,

$$\text{Volume of cuboid} = 3x^2 - 12x$$
$$l \times b \times h = 3x^2 - 12x$$
$$= 3x(x-4)$$
$$l \times b \times h = 3 \times x \times (x-4)$$

hence, length $= 3$, breadth $= x$, height $= (x-4)$.

Solve (ii) Given that,

$$\text{Volume of cuboid} = 12ky^2 + 8ky - 20k$$
$$l \times b \times h = 12ky^2 + 8ky - 20k$$
$$= 4k(3y^2 + 2y - 5)$$
$$= 4k(3y^2 - 3y + 5y - 5)$$
$$= 4k[3y(y-1) + 5(y-1)]$$
$$l \times b \times h = 4k \times (3y+5) \times (y-1)$$

V. Good
23/7/25

hence, length $= 4k$, breadth $= (3y+5)$, height $= (y-1)$.